Enhancing the convergence efficiency of a self-propelled agent system via a weighted model

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We investigate a weighted self-propelled agent system, wherein each agent's direction is determined by its spatial neighbors' directions with exponential weights according to the neighbor numbers. In order to describe the fact that some agents with more neighbors might have larger influence on its neighbors, we introduce a scaling exponent of the neighbor number between 0 and ∞. When the exponent is equal to 1, the convergence efficiency is enhanced in our simulation. Furthermore, as the exponent increases, i.e., the effect of weight becomes stronger, the network of agents becomes easier to achieve direction consensus.

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I. INTRODUCTION

Biological swarms are ubiquitous in nature, such as herds of quadrupeds, school of fish and flocking of birds, etc. From biology to physics, there has been a long standing interest in swarm research [1–4]. Reynolds created the first computer simulation of flocking with three heuristic rules known as cohesion, separation and alignment [5,6], and named it as Boid model. In 1995, Vicsek *et al.* performed research on the condition of alignment, and provided a simplified version of the Boid model [7]. In the Vicsek model, N self-propelled agents are driven toward different directions with a constant absolute velocity in a squared zone. At each time step, every agent updates its direction according to the average direction of agents' motion in its neighborhood of radius R. The simulation results [7] illustrate the fact when the density of the system is high and the noise is small enough, all agents will converge to the same direction on a macroscopic scale.

In recent years, as development of research in distributed sensor network [8], unmanned aerial vehicles [9], underwater vehicles [10] and attitude alignment for clusters of satellites, etc., Vicsek model as a fundamental model of consensus has triggered widespread interest. In order to obtain the theoretical explanations, Vicsek model is given in terms of a family of undirected graphs [11–18]. In the standard Vicsek model, the influencing radius and absolute velocity are invariable [7]. The influencing radius is randomly chosen according to a power-law distribution, which can make the network heterogeneous and enhance the convergence efficiency [19]. The agent updates its direction and speed adaptively according to the degree of consensus among its neighbors [20] or according to the local order parameter of consensus and the global consensus [21]. All the above methods and strategies are utilized to enhance the convergence efficiency.

At present, researches on complex networks demonstrate that networks are inhomogeneous [22–28], i.e., the degree of some nodes is much larger than the others. These nodes affect the structure and the dynamic process of network a lot. For instance, in the world wide web, the probability of connection increases with the degree for a new website; in the financial network and the traffic network, that removing some hub nodes may lead to a break down of the whole network; in the animal swarming, very few individuals are known to be able to influence the group [29]. In the Vicsek model, although influencing radius is the same, the neighbor number of each agent is different. Increasing the radius of some agents can make the network heterogeneous and enhance the convergence efficiency [19]. However, the influencing radius of each agent should be confined to a small range because the perception capability is limited for animals and cost as well as technology is restricted for actual multiagent systems (such as robot formation and moving sensor network).

In this paper, we propose a weighted self-propelled agent system, where the weight of each agent is determined by its number of neighbors. The direction of each agent is updated by the weighted average directions of its neighbors, instead of average directions. The aim of this work is to accelerate convergence and its degree of the self-propelled agent system. The simulation results demonstrate that the convergence time and the degree of consensus of the weighted model are significantly improved compared with the unweighted model. Furthermore, convergence efficiency in the self-propelled agent system is enhanced when the weight of the neighbor number is increased.

II. VICSEK MODEL

In the Vicsek model, a group of N agents are moving in a $L \times L$ square zone with the same and constant velocity but toward different directions. At initial time, the agents are randomly distributed, and their initial directions are also uniformly distributed in the interval $[0, 2\pi)$. At each time step,

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the direction of each agent is determined by the average directions of all the agents within a circle centered at the given agent, whose influencing radius is R. At time t, the position of a specific agent is updated according to

$$x_i(t+1) = x_i(t) + v_0 e^{i\theta_i(t)} \quad (1)$$

And its direction is updated as

$$e_i^{\theta}(t+1) = e^{i\Delta\theta_i(t)} \frac{\sum_{j\in\Gamma_i(t+1)} e^{i\theta_j(t)}}{\left\| \sum_{j\in\Gamma_i(t+1)} e^{i\theta_j(t)} \right\|_2}, \quad (2)$$

where $\Delta\theta_i$ denotes the white noise, $e^{i\theta_i(t)}$ denotes unit directional vector, and $\Gamma_i(t+1)$ is the set of neighbors for agent i at time step $t+1$.

In order to measure the degree of consensus for all the agents, an order parameter is introduced as [7,30]

$$V_\alpha = \frac{1}{N} \left\| \sum_{i=1}^{N} e^{i\theta_i(t)} \right\|_2, \quad 0 \le V_\alpha \le 1. \quad (3)$$

A larger value of V_α indicates a better consensus, especially when $V_\alpha=1$, all the agents moving in the same direction. Numerical stimulation claims that, in the circumstances of high density and low noise, all the agents will definitely approach to the consensus state, namely, having the same direction of velocity after definite time steps (convergence time) [31].

In this paper, two aspects of convergence efficiency are defined as

Convergence time t. When the direction θ of all the agents in the system become constant or the sum of change is small enough, the number of time steps have past is defined as convergence time.

Degree of consensus V_α. To evaluate the degree of direction consensus among agents, which is shown in Eq. (3).

In order to measure the fluctuations of the order parameter, the Binder cumulant G [14,18,32,33] is defined as

$$G = 1 - \frac{\langle V_\alpha^4 \rangle_t}{3\langle V_\alpha^2 \rangle_t^2}, \quad (4)$$

where $\langle \cdot \rangle_t$ indicates time average.

III. WEIGHTED MODEL

The *degree* is an essential variable in complex network as the neighbor number in the self-propelled agent system. The agent with more neighbors might have larger influences on its neighbors, which should play a very important role in the dynamic process. Thus, we introduce a weight related to the *degree* of each agent into Vicsek model when the directions are updated. Moreover, in order to diversify the difference between the large degree agents and the small degree agents, a general exponential weight model is proposed. The weight $\Upsilon_j^{(\lambda)}$ denotes an exponential form of the neighbor j of agent i at step t, and the mathematic description of weight is defined as

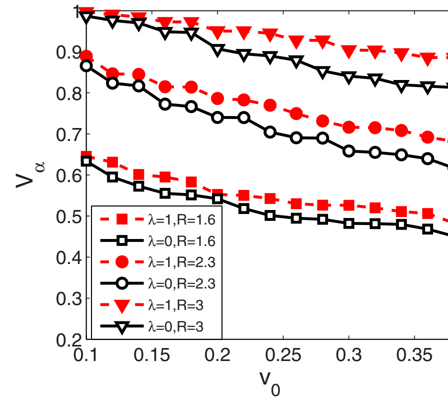

FIG. 1. (Color online) The degree of consensus V_α as a function of absolute velocity v_0 with changing R in cells of various values for $\lambda=0$ or $\lambda=1$. All the data points above are obtained by averaging over 300 different realizations.

$$\Upsilon_j^{(\lambda)}(t) = \frac{n_k^{\lambda}(t)}{\sum_{k\in\Gamma_j(t+1)} n_k^{\lambda}(t)}, \quad (5)$$

where $n_k(t)$ denotes the *degree* of agent k at time step t.

Introducing the weight to the direction updating equation Eq. (2), we will get the weighted model. In this model, each agent influences its neighbors' position according to the Eq. (1) and directions according to the following equations at time t,

$$e^{i\theta_i(t+1)} = \frac{\sum_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t) e^{i\theta_j(t)}}{\left\| \sum_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t) e^{i\theta_j(t)} \right\|_2}, \quad (6)$$

where $\Sigma_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t)=1$, and $\Upsilon_j^{(\lambda)} \ge 0$.

When $\lambda=0$, the new model is the same as the standard Vicsek model, where each agent in the system has the same weight.

As λ increases, the weight of the large degree agent becomes larger, and in the contrary, the weight of the small

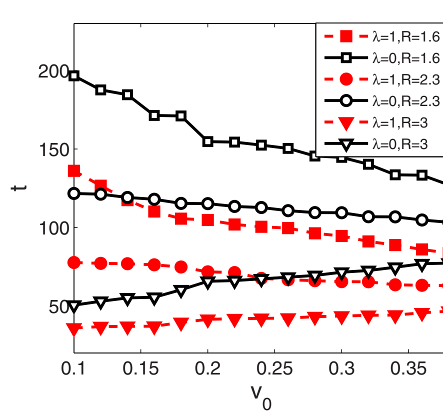

FIG. 2. (Color online) The convergence time t as a function of absolute velocity v_0 with changing R in cells of various values (1.6 to 3) for $\lambda=0$ and $\lambda=1$. All estimates are the results of averaging over 300 realizations.

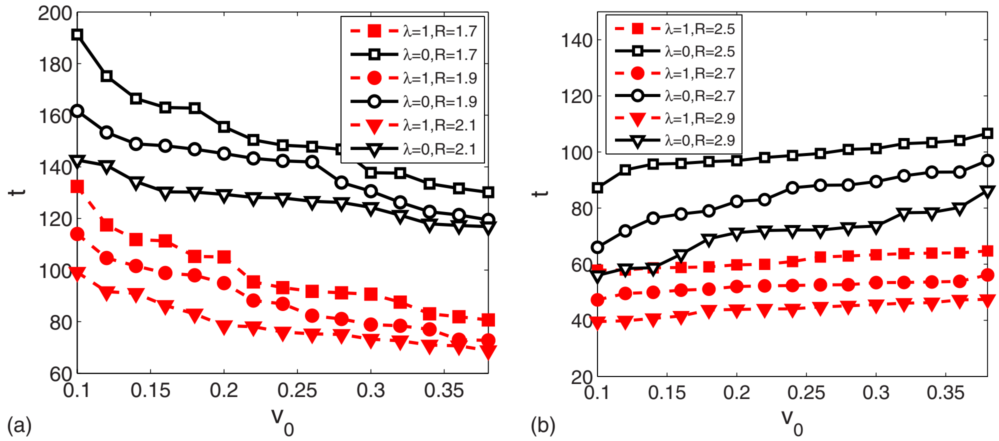

FIG. 3. (Color online) (a) Plots of the convergence time t on the absolute velocity v_0 for different λ and R various between 1.7 and 2.1. (b) Plots of the convergence time t on the velocity v_0 for different λ and R various between 2.5 and 2.9. All quantities are averaged over 300 realizations.

degree agent is smaller. Especially, when $\lambda=1$, it denotes that the weight of each agent is its *degree*.

When λ increases, the difference of weight between the large *degree* agent and the small *degree* agent enlarges, and the effect of *degree* as weight is amplified. Furthermore, When $\lambda \to +\infty$, an exponent infinity model is proposed as

$$e^{i\theta_i(t+1)} = \lim_{\lambda\to+\infty} \frac{\sum_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t) e^{i\theta_j(t)}}{\left\| \sum_{j\in\Gamma_i(t+1)} \Upsilon_j^{(\lambda)}(t) e^{i\theta_j(t)} \right\|_2} = \frac{\sum_{j\in\Omega_i(t+1,1)} e^{i\theta_j(t)}}{\left\| \sum_{j\in\Omega_i(t+1,1)} e^{i\theta_j(t)} \right\|_2}, \tag{7}$$

where $\Omega(t+1,1)$ denotes a set of agents which are the neighbors of agent i and with the largest degree among the neighbors of agent i.

The rule of updating direction in the exponent infinity model is as follows: the direction of each agent is determined by the average directions of agents who own the most neighbors among the circle with influencing radius and centered at the given agent at each time step illustrated in Eq. (7).

IV. SIMULATIONS AND DISCUSSIONS

In order to compare the effect of the standard model with the weighted model, we consider N agents moving in the two-dimensional (2D) plane. In our system we have open boundaries since we keep track only of the coordinates of each agent [20,34] instead of periodic boundary conditions [7]. The system consists of 200 agents positioned initially in a square of size 10×10. Due to the limitation of influencing radius R, each agent can only communicate with a certain part of agents in the range of its radius and change its direction according to this local information. The absolute velocity v_0 determines the changing frequency of neighboring agents. Different influencing radius and absolute velocities will give rise to diverse convergence time and degree of consensus, which will be numerically investigated in this paper.

Figure 1 shows the degree of consensus V_α as a function of the absolute velocity v_0 with different R and λ. For fixed R and λ, we can see that V_α is a decreasing function of v_0, because for a large v_0, each agent moves away from its neighbors before direction consensus. However, it decreases

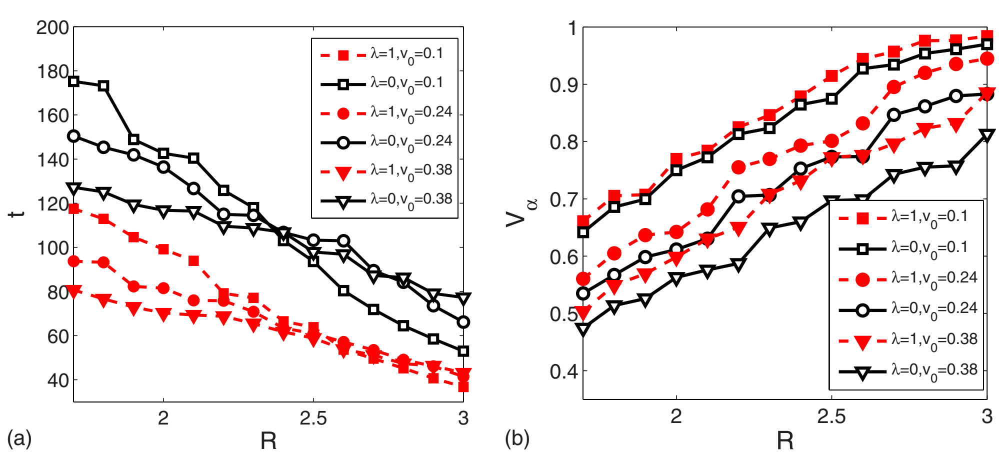

FIG. 4. (Color online) (a) Plots of the convergence time t on the R for different λ and v_0. (b) Plots of the degree of consensus V_α on R for different λ and v_0. All quantities are averaged over 300 realizations.

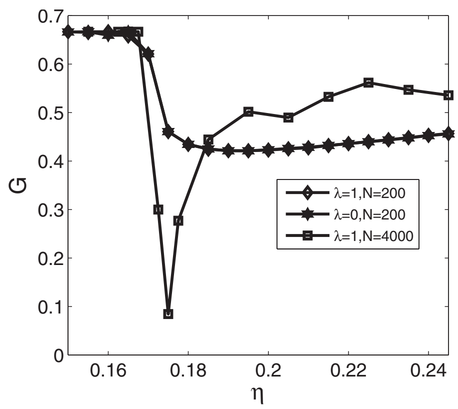

FIG. 5. The Binder cumulant G as a function of the noise amplitude η with various system size n. R is 2 in the simulation. The time averages have been computed over 1×10^4 time steps.

slower for larger value of λ, especially when R is large. It is also illustrated that larger R is advantageous to direction consensus in Fig. 1. Furthermore, for fixed R and v_0, the degree of consensus V_α is larger when λ=1 than when λ=0. Therefore, if the absolute velocity v_0 is large enough, even though it is very difficult or even practically impossible to achieve direction consensus in the original Vicsek model which corresponds to λ=0, the degree of consensus may still high in weighted model, which implies large λ is beneficial to degree of consensus.

The convergence time t as a function of absolute velocity v_0 with changing R in cells of various values [1.6 to 3] for λ=0,1 is shown in Fig. 2. Quite remarkably, when R is below a critical point [$R\leq2.3$ in Fig. 2] and λ is set to be 0 or 1, t is a decreasing function of v_0, but t would rise when R is equal to 3 as increasing v_0. In Fig. 1, it is known that the faster each agent moves, the more likely the system is to diffuse, and the larger the influencing radius R is, the more likely the system obtains consensus. Considering the opposite effects that R and v_0 work on the consensus of system, conclusions could be drawn from Fig. 2. When both of them are large or small, every agent would adjust its direction from time to time, so more convergence time is needed, partly because the system switches topology continuously. On the other hand, when R is small and v_0 is large, the system will quickly converge but diffuse, for both of the two factors are positive to diffusion. On the contrary, when R is large and v_0 is small, the system will quickly converge and reach consensus for both of the two factors could promote the degree of consensus. Figure 2 also shows that when R and v_0 are fixed, the convergence time is less for a larger value of λ. The proof is in the Appendix.

Figure 2 indicates there exists a critical point of R affecting the change direction of t. Thus, some further and detailed results are done in Fig. 3. From the results of Figs. 2 and 3 we can conclude that for a fixed λ when $R\leq2.3$ t is a decreasing function of v_0 [see in Fig. 3(a)] and when $R\geq2.4$, t is a increasing function of v_0 [see in Fig. 3(b)]. It is shown in Fig. 3(a) that when λ is kept constant, t decreases faster when R is smaller. And Fig. 3(b) illustrates that for a fixed λ, t increases slower when R is smaller.

The convergence time t and degree of consensus V_α as a function of R is shown in Fig. 4. The convergence time t is a decreasing function of R and V_α is a increasing function of R. In addition, for a smaller v_0, t decreases faster in Fig. 4(a). It also implies that the weighted model shortens the convergence time and enlarges the degree of consensus. Especially, when the v_0 is larger [$v_0=0.38$], the weighted model enlarges the degree of consensus more than when v_0 is smaller [$v_0=0.1$].

The noise amplitude η not only affects the degree of consensus but also the fluctuations of the order parameter G, which will be numerically investigated in Figs. 5 and 6. The Binder cumulant G as a function of the noise amplitude η is shown in Fig. 5. When $\eta>0.19$, G of large system size [$n=4000$] is larger than the small system size [$n=200$], because the density is larger for larger system size for fixed L in the simulation. The Binder cumulant G has a down peak for $N=4000$, which shows that there are large fluctuations of V_α when η is near η_c. The simulation results demonstrate that the curves of G for λ=0 and 1 collapse, so the fluctuations of V_α is the same, thus it makes possible for us to study the degree of consensus V_α as a function of the noise amplitude η, which is shown in Fig. 6. Clearly, when λ,R,v_0 are kept

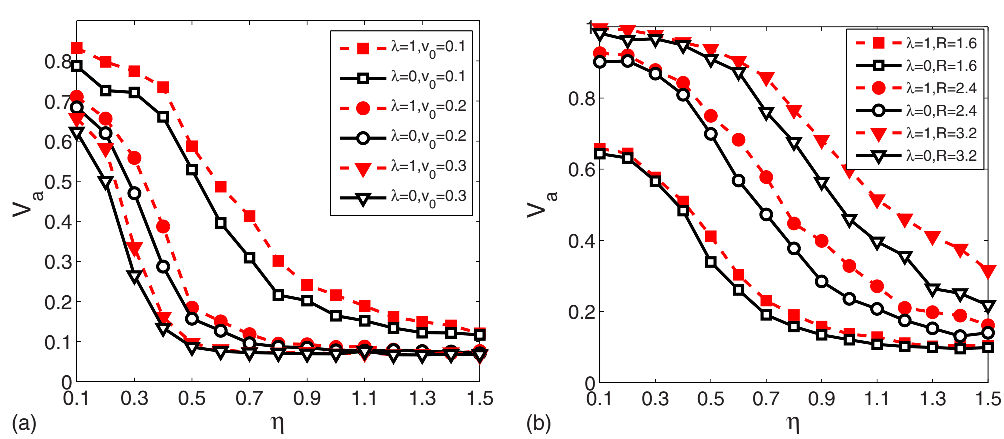

FIG. 6. (Color online) (a) The degree of consensus V_α, a function of the noise amplitude η with various absolute velocity v_0. R is 2 in the simulation. (b) The degree of consensus V_α, a function of the noise amplitude η with various R. Suppose v_0 is 0.1 in the simulation. All estimates are the results of averaging over 300 realizations.

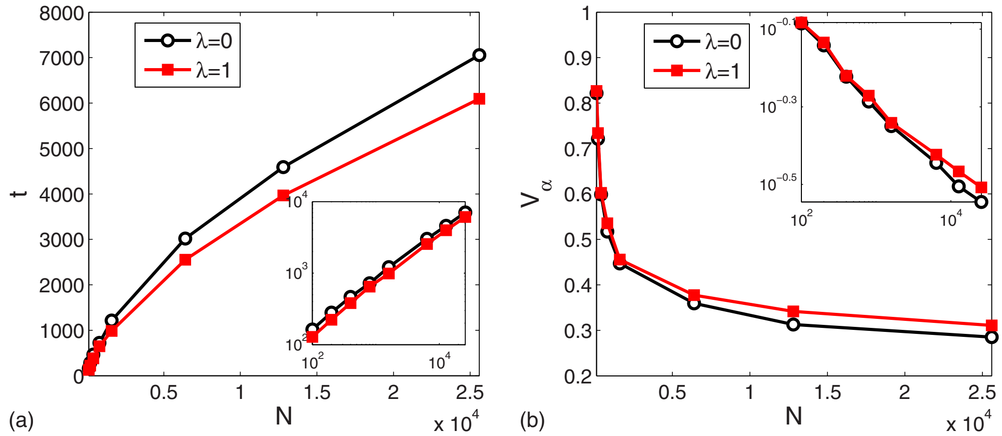

FIG. 7. (Color online) The convergence time t (a) and the degree of consensus V_α (b) as a function of the system size N for $\lambda=0$ and 1. The results shown in Fig. 7 were obtained from simulations in which $v_0=0.1$ and $R=2.0$. Data shown are the mean of 300 replicates.

constant V_α is decreasing function of the noise amplitude η. It implies that large λ leads to a better direction consensus for fixed v_0 and η [see in Fig. 6(a)] or for fixed R and η [see in Fig. 6(b)].

Finite size effects have been shown to play a very important role in the Vicsek model, thus the convergence time t and the degree of consensus V_α as a function of the system size N is shown in Fig. 7. when R is kept constant, we can see that for any value of system size N, the weighted model converges faster [see in Fig. 7(a)] and achieves a better direction consensus [see in Fig. 7(b)] than the original Vicsek model does. Figure 7(a) also demonstrates that the convergence time t is a power function of N which can be described as $t\sim N^\beta$, where $\beta=0.6454(\pm 0.0211)$ for the weighted model and $\beta=0.6353(\pm 0.0213)$ for the Vicsek model.

As is shown in Fig. 8(a), convergence time t decreases as the exponent λ or influencing radius R increases. In terms of a fixed exponent λ, we can see that t is a decreasing function of R, due to the fact that increasing R improves the connectivity of the network. Moreover, for any given value of the influencing radius R, t is a decreasing function of λ, implying that if the effect of *degree* as weight becomes stronger, it will be faster to obtain consensus. We also compute the degree of consensus when the self-propelled agent system gets to convergence. Figure 8(b) implies that the degree of consensus is increasing when the effect of *degree* as weight becomes stronger or network has better connectivity.

V. CONCLUSION

The collective dynamics of intelligent multiagents is not only a common phenomena in nature, but also a required in-depth investigation in engineering. Improving the convergence efficiency will bring in its extensive application. Recently, some scholars investigated the effect of the weighted coupling strength depending on the number of neighbors for the purpose of improving the efficiency of network synchronization on complex network [35]. However, utilizing the topology structure of mobile network to enhance the convergence efficiency is rarely reported. In this paper, we introduce the weight based on the size of neighborhood into the original Vicsek model. The simulation results show that this approach can accelerate consensus process and improve convergence efficiency. Furthermore, when the exponent is increasing, the contribution of the weight is larger, the self-

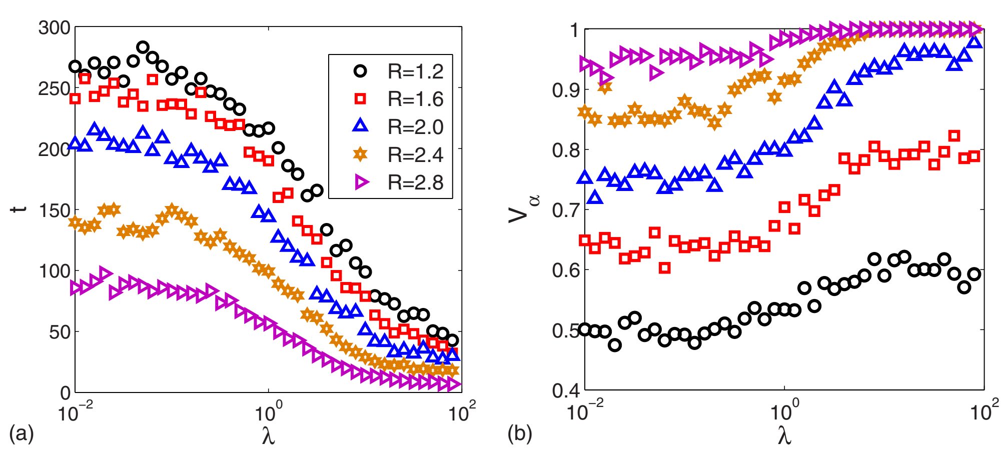

FIG. 8. (Color online) The convergence time t (a) and the degree of consensus V_α (b) as a function of the exponent λ for different influencing radius R. The results shown in Fig. 8 were obtained from simulations in which v_0 was set to be 0.1. Data shown are the mean of 300 replicates.

propelled agent system is much easier to obtain consensus even for the noise disturbance.

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APPENDIX

Here we prove that the weighted model accelerates the convergence of self-propelled agent system [36]. Our main assumption is that the system, which can be described by graphs and matrices, must be connected at every step. On the one hand, the Laplacian matrix of Vicsek model is $L=A-D$, where A is the adjacency matrix, D is a diagonal matrix and each diagonal element $d_i=\Sigma_{j=1}^{n}a_{ij}$. On the other hand the Laplacian matrix of the weighted model is $\hat{L}=\hat{A}-\hat{D}$, where $\hat{A}=\frac{1}{2}(AD^{\rho}+D^{\rho}A)$, $\hat{D}$ is a diagonal matrix and each diagonal element $\hat{d}_i=\Sigma_{j=1}^{n}\hat{a}_{ij}$.

As is known, the second large eigenvalue of Laplacian matrix is the algebraic connectivity which demonstrates the convergence of the system [12]. Without loss of generality, we suppose the eigenvalue of the Laplacian matrix of weighted model contents $\lambda_1(\hat{L})\geq\lambda_2(\hat{L})\geq\ldots\geq\lambda_n(\hat{L})$ and the eigenvalue of the Laplacian matrix of Vicsek model contents $\lambda_1(L)\geq\lambda_2(L)\geq\ldots\geq\lambda_n(L)$.

According to the peculiarity of Laplacian, it is known that $\lambda_1(L)=\lambda_1(\hat{L})=0$.

The weighted model converges faster, if the Eq. (A1) contents.

$$\lambda_2(L)\geq\lambda_2(\hat{L}). \tag{A1}$$

The Laplacian of the original Vicsek model is

$$L=\begin{bmatrix} -d_1 & a_{12} & \ldots & a_{1n} \\ a_{21} & -d_2 & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & -d_n \end{bmatrix}, \tag{A2}$$

and $L=[l_{ij}]_{n\times n}$, $\Sigma_{j=1}^{n}l_{ij}=\Sigma_{i=1}^{n}l_{ij}=0$.

The Laplacian of the weighted model is

$$\hat{L}=\begin{bmatrix} -\hat{d}_1 & \frac{1}{2}a_{12}(d_1^{\rho}+d_2^{\rho}) & \ldots & \frac{1}{2}a_{1n}(d_1^{\rho}+d_n^{\rho}) \\ \frac{1}{2}a_{21}(d_2^{\rho}+d_1^{\rho}) & -\hat{d}_2 & \ldots & \frac{1}{2}a_{2n}(d_2^{\rho}+d_n^{\rho}) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2}a_{n1}(d_n^{\rho}+d_1^{\rho}) & \frac{1}{2}a_{n2}(d_n^{\rho}+d_2^{\rho}) & \ldots & -\hat{d}_n \end{bmatrix}, \tag{A3}$$

and $\hat{L}=[\hat{l}_{ij}]_{n\times n}$, $\Sigma_{j=1}^{n}\hat{l}_{ij}=\Sigma_{i=1}^{n}\hat{l}_{ij}=0$.

Let Eq. (A2) minus Eq. (A3) be M, then $M=L-\hat{L}$, so

$$M=\begin{bmatrix} \hat{d}_1-d_1 & -\frac{1}{2}a_{12}(d_1^{\rho}+d_2^{\rho}-1) & \ldots & -\frac{1}{2}a_{1n}(d_1^{\rho}+d_n^{\rho}-1) \\ -\frac{1}{2}a_{21}(d_2^{\rho}+d_1^{\rho}-1) & \hat{d}_2-d_2 & \ldots & -\frac{1}{2}a_{2n}(d_2^{\rho}+d_n^{\rho}-1) \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2}a_{n1}(d_n^{\rho}+d_1^{\rho}-1) & -\frac{1}{2}a_{n2}(d_n^{\rho}+d_2^{\rho}-1) & \ldots & \hat{d}_n-d_n \end{bmatrix}=[m_{ij}]_{n\times n}. \tag{A4}$$

Because the system is connected, $d_i\geq 1$ for every agent, suppose the set of nodes connected with the node i is $j_1,j_2,\ldots,j_p,\ldots j_k$ $(d_i=k\geq 1)$. Then the ith diagonal element of the M matrix is

$$m_{ii}=\frac{1}{2}\left(k^{1+\rho}+\sum_{p=1}^{k}d_{j_p}^{\rho}\right)-k \tag{A5}$$

where $i=1,2,\ldots,n$.

So it is easy to get

$$\frac{dm_{ii}}{d\rho}=\frac{1}{2}\left(k^{1+\rho}\ln k+\sum_{p=1}^{k}d_{j_p}^{\rho}\ln d_{j_p}\right). \tag{A6}$$

There are two cases $k=1$ and $k\geq 2$. When $k=1$, without loss of generality, we suppose the node is connected with node j_s. If $d_{j_s}=1$, the whole graph is not connected, so $d_{j_s}\geq 2$, so $dm_{ii}/d\rho>0$. When $k\geq 2$, it is clearly that $dm_{ii}/d\rho>0$. Thus m_{ii} is an increasing function of ρ. Moreover, when $\rho=0$, $m_{ii}=0$. Thus we know that when $\rho>0$, $m_{ii}>0$ for any $i=1,2,\ldots,n$.

According to the Gersgorin disk theory, for any eigenvalue ζ of M,

$$|\zeta-m_{ii}|\leq\sum_{1\leq j\leq n,\ i\neq j}|m_{ij}|. \tag{A7}$$

Thus, we can get a conclusion that

$$\lambda_i(L) - \lambda_i(\hat{L}) \geq 0. \tag{A8}$$

So the weighted model converges faster than the original Vicsek model. Furthermore, it is easy to know $dm_{ii}/d\rho$ is an increasing function of ρ, which demonstrates that m_{ii} enlarge faster when ρ is larger. Then we know that enlarging ρ will accelerate convergence.

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